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ΔM_K and ε_K in SUSY at the Next-to-Leading order

Marco Ciuchini[1], **Livio Conti**[2], **Andrea Donini**[3], **Enrico Franco**[4], **Vicent Gimenez**[5], **Leonardo Giusti**[6], **Vittorio Lubicz**[1], **Guido Martinelli**[4], **Antonio Masiero**[7], **Ignazio Scimemi**[4], **Luca Silvestrini**[8], **Mauro Talevi**[9], **Anastassios Vladikas**[2]

[1]*Dip. di Fisica, Università di Roma Tre and INFN, Sezione di Roma III, Via della Vasca Navale 84, I-00146 Roma, Italy;* [2]*INFN, Sezione di Roma II, and Dip. di Fisica, Univ. di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Roma, Italy;* [3]*Dep. de Fisica Teorica, Univ. Autonoma Madrid, Fac. de Ciencias, C-XI, Cantoblanco, E-28049 Madrid, Spain;* [4]*Dipartimento di Fisica, Università di Roma "La Sapienza" and INFN, Sezione di Roma, P.le A. Moro, I-00185 Roma, Italy;* [5]*Dep. de Fisica Teorica and IFIC, Univ. de Valencia, Dr. Moliner 50, E-46100, Burjassot, Valencia, Spain;* [6]*Scuola Normale Superiore, P.zza dei Cavalieri 7 and INFN, Sezione di Pisa, 56100 Pisa, Italy;* [7]*SISSA-ISAS, Via Beirut 2-4, I-34013 Trieste, Italy and INFN, Sezione di Trieste, Trieste, Italy;* [8]*Physik Department, Technische Universität München, D-85748 Garching, Germany;* [9]*Department of Physics & Astronomy, University of Edinburgh, The King's Buildings, Edinburgh EH9 3JZ, UK*

Abstract: We perform a Next-to-Leading order analysis of $\Delta S = 2$ processes beyond the Standard Model. Combining the recently computed NLO anomalous dimensions and the B parameters of the most general $\Delta S = 2$ effective Hamiltonian, we give an analytic formula for ΔM_K and ε_K in terms of the Wilson coefficients at the high energy scale. This expression can be used for any extension of the Standard Model with new heavy particles. Using this result, we consider gluino-mediated contributions to $\Delta S = 2$ transitions in general SUSY models and provide an improved analysis of the constraints on off-diagonal mass terms between the first two generations of down-type squarks. Finally, we improve the constraints on R-violating couplings from ΔM_K and ε_K.

Keywords: Supersymmetric Standard Model, Kaon Physics, CP violation, NLO Computations.

Contents

1. Introduction

The prescription of minimality in the number of new particles introduced to supersymmetrise the Standard Model (SM), together with the demand of conservation of baryon and lepton numbers, does not prevent the appearance of more than 100 new SUSY parameters in addition to the 18 already present in the SM. Fortunately for SUSY predictivity, most of this enormous parameter space is already phenomenologically ruled out. Flavour changing neutral current (FCNC) and CP violating phenomena are the protagonists of this drastic reduction of the SUSY degrees of freedom. Therefore, the interest on the FCNC constraints on SUSY is well motivated: given the large degree of arbitrariness in the construction of low-energy SUSY models, they represent one of the main criteria we have for selecting viable theories. At the same time, they can shed some, albeit dim, light on the more fundamental physics from which low-energy SUSY stems.

The ignorance of the underlying physics compels us to find ways to analyse the impact of FCNC and CP violating processes on low-energy SUSY in a model-independent manner. Since the pioneering work of Hall, Kostelecky and Raby in 1986 [1], the most adequate tool for such analyses has proven to be the so-called mass insertion approximation. A brief description of this method will be provided later on. We note here that this method has been applied, so far, mainly to the most genuine SUSY sources of FCNC, i.e. the Flavour Changing (FC) couplings of gluinos and neutralinos to fermions and sfermions [2]. Differently from the other sources of FCNC in SUSY (namely W, charged Higgs and chargino exchanges), these couplings have no analogue in the SM.

As long as we deal with general squark mass matrices, the inclusion of gluino-mediated FCNC diagrams alone is sufficient to get the correct bulk of the SUSY contribution. In specific models, corresponding to particularly restricted squark masses, it may occur, however, that other contributions become more important. This is the case, for example, of the Constrained Minimal Supersymmetric Standard Model (CMSSM) with exact universality of the soft breaking terms: in this case, in regions of the SUSY parameter space where charginos, stops and/or charged Higgs are relatively light, their exchange may overwhelm the gluino contribution to FCNC (see for instance [3]). In this work we intend to study the general case of squark mass matrices and, thus, we concentrate on the gluino and squark exchange contributions. We postpone the inclusion of the chargino-squark contributions in the mass-insertion framework to a subsequent work.

In the literature there exist four analyses which make use of the abovementioned method to perform a complete study of the whole variety of FCNC processes in the hadronic and leptonic sectors [3]–[6]. These studies do not include QCD radiative corrections and make use of the Vacuum Insertion Approximation (VIA) in the evaluation of the hadronic matrix elements. A major step forward was recently made by the inclusion of the leading QCD corrections in the evaluation of the gluino-exchange contributions to K^0–$\bar{K}^0$ mixing [7]. What Bagger et al. found is that the leading QCD corrections affect the results in a non-negligible way: in particular for comparable values of the squark and gluino masses, the QCD corrections increase the lower bound on the squark masses of the first two generations by a factor three. As for the B-parameters of the hadronic matrix elements, the VIA values were used.

Motivated by the encouraging results of ref. [7], by the recent computation of the Next-to-Leading (NLO) QCD corrections to the most general $\Delta F = 2$ effective Hamiltonian $\mathcal{H}_{\rm eff}^{\Delta F=2}$ [8] and by the lattice calculation of the full set of B-parameters contributing to the K^0 mixing matrix elements [9, 10], we present here the results of a new analysis of K^0–$\bar{K}^0$ mixing which includes these progresses intended to improve the theoretical accuracy.

A full NLO computation needs the evaluation of the $O(\alpha_s)$ corrections to the Wilson coefficients at the scale of the SUSY masses running in the loop. This piece of information is not available yet for gluino contributions.[1] We can only invoke the smallness of α_s evaluated at such large scale as a (arguable) hint for the relative smallness of these corrections. Even with this limitation, we think that it is relevant to provide the improvement that the knowledge of the NLO QCD corrections to $\mathcal{H}_{\rm eff}^{\Delta S=2}$ and the lattice values of the B-parameters allow us to obtain.

A final comment is in order. One may wonder whether such a refinement (inclusion of NLO QCD corrections and B-parameters from lattice computations) is worth in a

[1]NLO QCD corrections to the matching conditions for charged-Higgs and chargino contributions to ε_K have been very recently computed [11].

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study where many uncertainties, due to the presence of undetermined SUSY parameters and to the use of certain approximations (in particular the omission of the full set of FCNC contributions with possible non-negligible interference effects), are present. We think that the answer is positive at least for two reasons. The practical one is that, due to the complete model-independence of our approach, it is possible to "test" rapidly the impact of FCNC processes on new classes of low-energy SUSY models by just comparing their predictions for the FC parameters δ (see below) with the constraints that we provide. Obviously a determination as precise as possible of these δs is welcome. The second motivation is that with the future inclusion of the chargino exchange contributions to FCNC processes we will be able to obtain a comprehensive framework where to perform general studies of low-energy SUSY. In view of this, the use of $\mathcal{H}_{\rm eff}^{\Delta F=2}$ consistently renormalised at the NLO and a better determination of the B-parameters play a major role in obtaining an efficient tool of analysis. Our aim is to approach as much as possible the situation of the computation of $\Delta F = 2$ processes in the SM, where the complete LO [12] and NLO [13] analyses have been carried out.

To this aim, we provide an analytic formula for the NLO $\mathcal{H}_{\rm eff}^{\Delta S=2}$ in terms of the Wilson coefficients at the SUSY scale. This expression, together with the values of the B-parameters, can be readily used to compute ΔM_K and ε_K in any given SUSY model. Indeed, the formula we give is valid in any extension of the SM in which new heavy particles contributing to FCNC processes are present, since the basis of operators considered here is the most general for $\Delta S = 2$ transitions. One just has to plug in the initial conditions computed in his favourite model. At the moment, the full NLO expression for $\mathcal{H}_{\rm eff}^{\Delta S=2}$, including the $O(\alpha_s)$ contributions to the matching conditions, is only available for the SM [13]. The contributions to ε_K (i.e. to the imaginary part of the Hamiltonian) are also known for the two Higgs doublets model [14] and for the chargino contribution in the CMSSM [11]. In all these cases, the $O(\alpha_s)$ terms in the matching amount to a very small correction, and the bulk of the effect is due to the running down to the hadronic scale. In our case, we find that the effect of NLO corrections to the running of the Left-Right operators is quite large (of the order of 30% or more), which is somewhat worrying with respect to the convergence of the renormalisation-group improved perturbative expansion.

The paper is organised as follows. Next section will introduce the $\mathcal{H}_{\rm eff}^{\Delta S=2}$ in SUSY with the inclusion of the NLO QCD corrections in the evolution from the SUSY scale down to low energy. We will also provide the expression of the Wilson coefficients at the hadronic scale (to be detailed below) as a function of the Wilson coefficients and of α_s at the SUSY scale. Section 3 deals with the evaluation of the hadronic matrix elements of the local operators of $\mathcal{H}_{\rm eff}^{\Delta S=2}$. Here we will replace the "traditional" values of the B-parameters in the VIA with the values which were recently obtained in a lattice computation [9]. Our quantitative results are presented in Sect. 4 and summarised in the Tables 2–7. These latter contain new constraints on the FC parameters δ and a comparison with the previous results. The major effect of our improved computation

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is felt by Left-Right operators. As a further application, in Sect. 5, we consider the contribution to K^0–$\bar{K}^0$ mixing in models with explicit R parity and lepton number violations. We obtain new bounds for the relevant R parity breaking λ'-type couplings and compare them to those which were previously obtained in the VIA case and without QCD corrections. Finally Sect. 6 contains some conclusions and an outlook.

2. Effective Hamiltonian for $\Delta S = 2$ processes in SUSY

In this Section, we describe the computation of gluino-mediated contributions to $\mathcal{H}_{\rm eff}^{\Delta S=2}$ at the NLO in QCD.

In ref. [7], $\mathcal{H}_{\rm eff}^{\Delta S=2}$ was computed at the LO in three different cases: i) $m_{\tilde{q}} \sim m_{\tilde{g}}$, ii) $m_{\tilde{q}} \ll m_{\tilde{g}}$ and iii) $m_{\tilde{q}} \gg m_{\tilde{g}}$, where $m_{\tilde{q}}$ is the average squark mass and $m_{\tilde{g}}$ is the gluino mass.

Case ii), in which $m_{\tilde{q}} \ll m_{\tilde{g}}$, cannot be realized in the framework considered here, in which the soft SUSY breaking terms are introduced at the Planck scale. This is due to the fact that the evolution from the Planck to the electroweak scale forbids such a mass hierarchy. In fact, neglecting the effects of Yukawa couplings and A-terms, one obtains in the down sector the following approximate expression for the ratio $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ at the electroweak scale, in terms of the value x_0 at the Planck scale [15]:

$$x \simeq \frac{9x_0}{1+7x_0} \longrightarrow \frac{9}{7}\,. \tag{2.1}$$

Even if one starts at the superlarge scale with an extreme hierarchy between squark and gluino masses ($x_0 \gg 1$), at the electroweak scale the two masses will be of the same order. For this reason, we will not consider the case $m_{\tilde{q}} \ll m_{\tilde{g}}$ in our analysis. Case iii), in which $m_{\tilde{q}} \gg m_{\tilde{g}}$, can be realized in some special models, such as effective supersymmetry [16] or models with a light gluino [17]. However, due to the peculiar features and signatures of these models, a careful analysis of these cases would lie beyond the scope of this paper (see ref. [18] for a NLO analysis of $\Delta S = 2$ processes in these models). Therefore, in the following we will consider the case $m_{\tilde{q}} \sim m_{\tilde{g}}$, and compare our results with the zeroth- [6] and leading-order [7] results previously published.

We will perform our computation in the so-called mass insertion approximation [1]. One chooses the super-CKM basis for the fermion and sfermion states, where all the couplings of these particles to neutral gauginos are flavour diagonal, while the genuine SUSY FC effect is exhibited by the non-diagonality of the sfermion mass matrices. Denoting by Δ^2 the off-diagonal terms in the sfermion mass matrices (i.e. the mass terms relating sfermions of the same electric charge, but different flavour), the sfermion propagators can be expanded as a series in terms of $\delta = \Delta^2/\tilde{m}^2$, where $\tilde{m}$ is the average sfermion mass. As long as Δ^2 is significantly smaller than $\tilde{m}^2$, we can just take the first term of this expansion and, then, the experimental information concerning FCNC and CP violating phenomena translates into upper bounds on the δs [4]–[6].

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Obviously the above mass insertion method presents the major advantage that one does not need the full diagonalisation of the sfermion mass matrices to perform a test of the SUSY model under consideration in the FCNC sector. It is enough to compute ratios of the off-diagonal over the diagonal entries of the sfermion mass matrices and compare the results with the general bounds on the δs which we provide here from ΔM_K and ε_K. This formulation of the mass insertion approximation in terms of the parameters δ is particularly suitable for model-independent analyses, but involves two further assumptions: the smallness of the off-diagonal mass terms with respect to the diagonal ones, and the degeneracy of the diagonal mass terms in the super-CKM basis. The latter assumption, related to the use of the average squark mass $\tilde{m}$, is well justified in our case, since, on quite general grounds, one does not expect a sizeable non-degeneracy of the first two generations of down-type squarks. It is also possible, however, to define the mass insertion approximation in a more general way, which is also valid for non-degenerate diagonal mass terms (see ref. [19] for details).

There exist four different Δ mass-insertions connecting flavours d and s along a sfermion propagator: $\left(\Delta^d_{12}\right)_{LL}$, $\left(\Delta^d_{12}\right)_{RR}$, $\left(\Delta^d_{12}\right)_{LR}$ and $\left(\Delta^d_{12}\right)_{RL}$. The indices L and R refer to the helicity of the fermion partners. The amplitude for $\Delta S = 2$ transitions in the full theory at the LO is given by the computation of the diagrams in fig. 1.

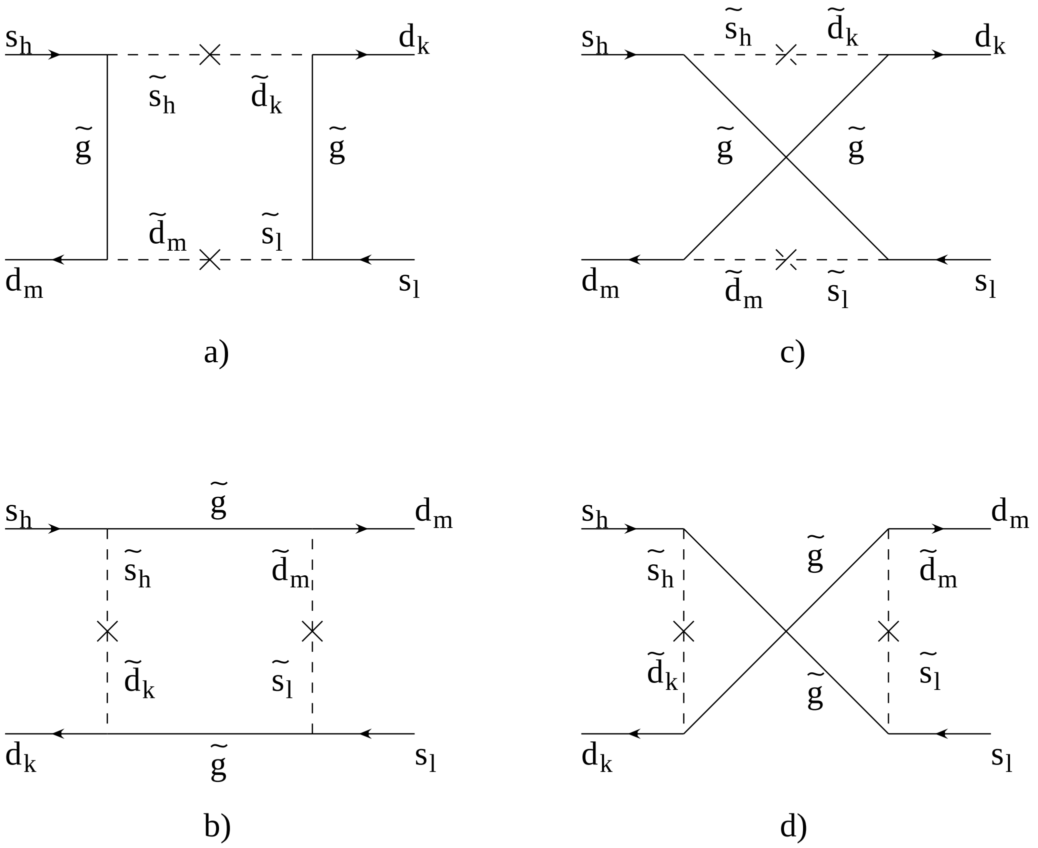

Figure 1: Feynman diagrams for $\Delta S = 2$ transitions, with $h, k, l, m = \{L, R\}$.

Having calculated the amplitude from the diagrams in fig. 1, one has to choose a basis of local operators and perform the matching of the full theory to the one described by $\mathcal{H}_{\rm eff}^{\Delta S=2}$. We have adopted the form

$$\mathcal{H}_{\rm eff}^{\Delta S=2} = \sum_{i=1}^{5} C_i\, Q_i + \sum_{i=1}^{3} \tilde{C}_i\, \tilde{Q}_i\,, \tag{2.2}$$

where

$$\begin{aligned} Q_1 &= \bar{d}^\alpha_L \gamma_\mu s^\alpha_L \bar{d}^\beta_L \gamma^\mu s^\beta_L\,, \\ Q_2 &= \bar{d}^\alpha_R s^\alpha_L \bar{d}^\beta_R s^\beta_L \\ Q_3 &= \bar{d}^\alpha_R s^\beta_L \bar{d}^\beta_R s^\alpha_L \\ Q_4 &= \bar{d}^\alpha_R s^\alpha_L \bar{d}^\beta_L s^\beta_R \\ Q_5 &= \bar{d}^\alpha_R s^\beta_L \bar{d}^\beta_L s^\alpha_R \end{aligned} \tag{2.3}$$

and the operators $\tilde{Q}_{1,2,3}$ are obtained from the $Q_{1,2,3}$ by the exchange $L \leftrightarrow R$. Here $q_{R,L} = P_{R,L}\, q$, with $P_{R,L} = (1 \pm \gamma_5)/2$, and α and β are colour indices.

At the lowest order in QCD, we obtain the following results for the Wilson coefficients:

$$\begin{aligned} C_1 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\left(24\, x\, f_6(x) + 66\, \tilde{f}_6(x)\right)\left(\delta^d_{12}\right)^2_{LL} \\ C_2 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\, 204\, x\, f_6(x)\left(\delta^d_{12}\right)^2_{RL} \\ C_3 &= \frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\, 36\, x\, f_6(x)\left(\delta^d_{12}\right)^2_{RL} \\ C_4 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\Bigg[\left(504\, x\, f_6(x) - 72\, \tilde{f}_6(x)\right)\left(\delta^d_{12}\right)_{LL}\left(\delta^d_{12}\right)_{RR} - \\ &\qquad - 132\, \tilde{f}_6(x)\left(\delta^d_{12}\right)_{LR}\left(\delta^d_{12}\right)_{RL}\Bigg] \\ C_5 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\Bigg[\left(24\, x\, f_6(x) + 120\, \tilde{f}_6(x)\right)\left(\delta^d_{12}\right)_{LL}\left(\delta^d_{12}\right)_{RR} - \\ &\qquad - 180\, \tilde{f}_6(x)\left(\delta^d_{12}\right)_{LR}\left(\delta^d_{12}\right)_{RL}\Bigg] \\ \tilde{C}_1 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\left(24\, x\, f_6(x) + 66\, \tilde{f}_6(x)\right)\left(\delta^d_{12}\right)^2_{RR} \\ \tilde{C}_2 &= -\frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\, 204\, x\, f_6(x)\left(\delta^d_{12}\right)^2_{LR} \\ \tilde{C}_3 &= \frac{\alpha_s^2}{216 m_{\tilde{q}}^2}\, 36\, x\, f_6(x)\left(\delta^d_{12}\right)^2_{LR}\,, \end{aligned} \tag{2.4}$$

where $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ and the functions $f_6(x)$ and $\tilde{f}_6(x)$ are given by:

$$
\begin{aligned}
f_6(x) &= \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5} \\
\tilde{f}_6(x) &= \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5} \,.
\end{aligned} \tag{2.5}
$$

In the absence of the $O(\alpha_s)$ corrections to the matching, we interpret the C_i given above as coefficients computed at the large energy scale $M_S \sim m_{\tilde{q}} \sim m_{\tilde{g}}$, i.e. $C_i \equiv C_i(M_S)$.

The Next-to-Leading anomalous dimension matrix for the most general $\mathcal{H}_{\rm eff}^{\Delta F=2}$ has been recently computed [8]. We use the Regularisation-Independent anomalous dimension in the Landau gauge (LRI), since we will make use of matrix elements computed in lattice QCD with the same choice of renormalisation scheme (see ref. [8] for details on the computation).

A full NLO computation would also require the $O(\alpha_s)$ corrections to the matching conditions in eq. (2.4). Unfortunately, such corrections are not available yet. One might argue that, being of order $\alpha_s(M_S)$, these contributions should be small, as suggested by the cases of the SM and of the two Higgs doublet model; however, this statement can only be confirmed by an explicit computation. Unfortunately, due to the absence of $O(\alpha_s)$ corrections to the matching, our $\mathcal{H}_{\rm eff}^{\Delta F=2}$ will be affected by a residual scheme dependence, which would be cancelled by the missing terms of order $\alpha_s(M_S)$.

Eq. (2.4) is obtained by integrating out all SUSY particles simultaneously. We then have to evolve the coefficients down to the hadronic scale $\mu = 2$ GeV, at which we have evaluated the matrix elements. The SM contribution can be computed independently and evolved from M_W to μ using the well-known NLO QCD corrections [13].

Constants	Values
M_K	497.67 MeV
f_K	159.8 MeV
m_d(2GeV)	7 MeV
m_s(2GeV)	125 MeV
m_c	1.3 GeV
m_b	4.3 GeV
m_t	175 GeV
$\alpha_s(M_Z)$	0.119

Table 1: Constants used in the phenomenological analysis.

We give here an analytic formula for the expression of the Wilson coefficients at the scale $\mu = 2$ GeV as a function of the initial conditions at the SUSY scale $C(M_S)$ and of $\alpha_s(M_S)$. This formula has been obtained by using the values in Table 1 for the SM parameters.

For $M_S > m_t$ we obtain

$$
C_r(\mu) = \sum_i \sum_s \left(b_i^{(r,s)} + \eta\, c_i^{(r,s)} \right) \eta^{a_i}\, C_s(M_S)\,, \tag{2.6}
$$

where, in the evolution of the coefficients from M_S, we have chosen $M_S = (M_{\tilde{g}} + M_{\tilde{q}})/2$.

$\eta = \alpha_s(M_S)/\alpha_s(m_t)$, $\mu = 2$ GeV and the magic numbers are given below:

$$a_i = (0.29, -1.1, 0.14, -0.69, 0.79)$$

$$\begin{array}{ll}
b_i^{(11)} = (0.82, 0, 0, 0, 0) & c_i^{(11)} = (-0.016, 0, 0, 0, 0) \\
b_i^{(22)} = (0, 0.061, 0.82, 0, 0) & c_i^{(22)} = (0, -0.013, 0.018, 0, 0) \\
b_i^{(23)} = (0, -0.092, 0, 0, 0) & c_i^{(23)} = (0, 0.013, 0.0078, 0, 0) \\
b_i^{(32)} = (0, -2.9, 0.34, 0, 0) & c_i^{(32)} = (0, 0.69, 0.012, 0, 0) \\
b_i^{(33)} = (0, 4.4, 0, 0, 0) & c_i^{(33)} = (0, -0.68, 0.0055, 0, 0) \\
b_i^{(44)} = (0, 0, 0, 2.4, -0.003) & c_i^{(44)} = (0, 0, 0, -0.23, 0.0026) \\
b_i^{(45)} = (0, 0, 0, 4.6, -0.38) & c_i^{(45)} = (0, 0, 0, -1.1, -0.041) \\
b_i^{(54)} = (0, 0, 0, -0.0024, 0.0035) & c_i^{(54)} = (0, 0, 0, 0.00034, -0.0012) \\
b_i^{(55)} = (0, 0, 0, -0.0045, 0.44) & c_i^{(55)} = (0, 0, 0, 0.0016, 0.018)
\end{array} \tag{2.7}$$

and we have only written the non-vanishing entries. The magic numbers for the evolution of $\tilde{C}_{1-3}$ are the same as the ones for the evolution of C_{1-3}. Formulae (2.6) and (2.7) can be used in connection with the B-parameters evaluated at $\mu = 2$ GeV, given in eq. (3.3), to compute the contribution to ΔM_K and ε_K at the NLO in QCD for any model of new physics in which the new contributions with respect to the SM originate from extra heavy particles. One just has to plug in the expression for the C_i evaluated at the large energy scale M_S in his favourite model. When the $O(\alpha_s)$ corrections to the $C_i(M_S)$ are available, one can obtain a full NLO, regularisation-independent result; in the cases where this corrections have not been computed yet, the results contain a residual systematic uncertainty of order $\alpha_s(M_S)$. We note that, due to the presence of large entries in the NLO anomalous dimension matrix, a systematic uncertainty of a few percents is present in the QCD evolution from the SUSY scale to the hadronic one. For this reason we only present the significant figures for the magic numbers.

3. Hadronic matrix elements

The matrix elements of the operators Q_i between K mesons in the VIA are given by:

$$\begin{aligned}
\langle K^0|Q_1|\bar{K}^0\rangle_{\rm VIA} &= \frac{1}{3} M_K f_K^2 \\
\langle K^0|Q_2|\bar{K}^0\rangle_{\rm VIA} &= -\frac{5}{24}\left(\frac{M_K}{m_s+m_d}\right)^2 M_K f_K^2 \\
\langle K^0|Q_3|\bar{K}^0\rangle_{\rm VIA} &= \frac{1}{24}\left(\frac{M_K}{m_s+m_d}\right)^2 M_K f_K^2 \\
\langle K^0|Q_4|\bar{K}^0\rangle_{\rm VIA} &= \left[\frac{1}{24}+\frac{1}{4}\left(\frac{M_K}{m_s+m_d}\right)^2\right] M_K f_K^2 \\
\langle K^0|Q_5|\bar{K}^0\rangle_{\rm VIA} &= \left[\frac{1}{8}+\frac{1}{12}\left(\frac{M_K}{m_s+m_d}\right)^2\right] M_K f_K^2 \,,
\end{aligned} \tag{3.1}$$

where M_K is the mass of the K^0 meson and m_s and m_d are the masses of s and d quarks respectively. Here and in the following, the same expressions of the B-parameters of the operators Q_{1-3} are valid for the operators $\tilde{Q}_{1-3}$, since strong interactions preserve parity.

In the case of the renormalised operators, we define the B-parameters as follows:

$$\begin{aligned}
\langle \bar{K}^0|\hat{Q}_1(\mu)|K^0\rangle &= \frac{1}{3} M_K f_K^2 B_1(\mu) \\
\langle \bar{K}^0|\hat{Q}_2(\mu)|K^0\rangle &= -\frac{5}{24}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K f_K^2 B_2(\mu) \\
\langle \bar{K}^0|\hat{Q}_3(\mu)|K^0\rangle &= \frac{1}{24}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K f_K^2 B_3(\mu) \\
\langle \bar{K}^0|\hat{Q}_4(\mu)|K^0\rangle &= \frac{1}{4}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K f_K^2 B_4(\mu) \\
\langle \bar{K}^0|\hat{Q}_5(\mu)|K^0\rangle &= \frac{1}{12}\left(\frac{M_K}{m_s(\mu)+m_d(\mu)}\right)^2 M_K f_K^2 B_5(\mu)\,,
\end{aligned} \tag{3.2}$$

where the notation $\hat{Q}_i(\mu)$ (or simply $\hat{Q}_i$) denotes the operators renormalised at the scale μ.

A few words of explanation are necessary at this point. The B-parameter of the matrix element $\langle \bar{K}^0|Q_1|K^0\rangle$, commonly known as B_K, has been extensively studied on the lattice due to its phenomenological relevance [20], and used in many phenomenological studies [21]. For the other operators, instead, all the phenomenological analyses beyond the SM have used B-parameters equal to one, which in some cases, as will be shown below, is a very crude approximation.

In eq. (3.2) the operators and the quark masses are renormalised in the same scheme (RI, $\overline{MS}$, etc.) at the scale μ and the numerical results for the B-parameters, $B_i(\mu)$, presented below refer to the Landau RI scheme. Moreover, without loss of generality, we have omitted terms which are of higher order in the chiral expansion, see eqs. (3.1), and which are usually included in the definition of the B-parameters (see ref. [9] for a thorough discussion of the advantages of our definition of the B-parameters).

In our numerical study, we have used, for $\mu = 2$ GeV:

$$\begin{aligned}
B_1(\mu) &= 0.60(6) \qquad (3.3)\\
B_2(\mu) &= 0.66(4) \\
B_3(\mu) &= 1.05(12) \\
B_4(\mu) &= 1.03(6) \\
B_5(\mu) &= 0.73(10)\,.
\end{aligned}$$

The central value we use for $B_1 = B_K$ corresponds to $B_K^{\overline{MS}}(2\,\mathrm{GeV}) = 0.61$, in agreement with the recent estimates of refs. [20, 22]. B_{2-5} have been taken from ref. [9], where all details of the computation can be found (for another determination of these B-parameters, calculated with perturbative renormalization, see ref. [10]).

4. Numerical analysis of $\Delta S = 2$ processes

We now present the results of a model-independent analysis of K^0–$\bar{K}^0$ mixing, for the case $m_{\tilde{q}} \sim m_{\tilde{g}}$. The K_L–K_S mass difference ΔM_K and the CP-violating parameter ε_K are given by:

$$\Delta M_K = 2\,\mathrm{Re}\langle K^0|\mathcal{H}_{\mathrm{eff}}^{\Delta S=2}|\bar{K}^0\rangle \tag{4.1}$$

$$\varepsilon = \frac{1}{\sqrt{2}\Delta M_K}\,\mathrm{Im}\langle K^0|\mathcal{H}_{\mathrm{eff}}^{\Delta S=2}|\bar{K}^0\rangle\,. \tag{4.2}$$

The SUSY (gluino-mediated) contribution to the low-energy $\mathcal{H}_{\mathrm{eff}}^{\Delta S=2}$ contains two real and four complex unknown parameters: $m_{\tilde{q}}$, $m_{\tilde{g}}$, $\left(\delta^d_{12}\right)_{LL}$, $\left(\delta^d_{12}\right)_{LR}$, $\left(\delta^d_{12}\right)_{RL}$ and $\left(\delta^d_{12}\right)_{RR}$. These parameters can be determined once a specific SUSY model is chosen, and the NLO contribution to ΔM_K and ε_K can be directly computed by using the expression for the low-energy $\mathcal{H}_{\mathrm{eff}}^{\Delta S=2}$ and the B-parameters given in eqs. (2.4), (2.6) and (3.3).

However, it is also useful to provide a set of model-independent constraints on the individual δ-parameters, obtained by neglecting the interference between the different SUSY contributions. This is justified (a posteriori) by noting that the constraints on different δ-parameters in the Kaon case exhibit a hierarchical structure, and therefore interference effects between different contributions would require a large amount of fine tuning.

The model-independent constraints are obtained by imposing that the sum of the SUSY contributions proportional to a single δ-parameter and of the SM contributions to ΔM_K and ε_K does not exceed the experimental value for these quantities.

A comment on the SM contributions is necessary at this point. Let us first consider ΔM_K. The short-distance SM contribution to this process is dominated by charm-quark exchange. Hence its value can be computed once the CKM elements V_{cd} and V_{cs} are determined. Making the very reasonable hypothesis that SUSY contributions to tree-level weak decays are negligible, the determination of the CKM elements V_{cd} and V_{cs} is unaffected by SUSY, and therefore the SM contribution to ΔM_K can be computed even in the presence of large SUSY contributions to loop processes. We are not considering here long-distance contributions to ΔM_K. The constraints we obtain from ΔM_K are therefore more conservative if the long-distance SM contributions add up to the short-distance ones.

On the other hand, the SM contribution to ε_K depends on the phase in the CKM matrix. This phase is usually extracted, in the context of the SM, from the analysis of ε_K and B^0–$\bar{B}^0$ mixing. However, in the presence of large (unknown) SUSY contributions to K^0–$\bar{K}^0$ and B^0–$\bar{B}^0$ mixing, the extraction of the CKM phase is not possible.

We therefore treat it as a free parameter. Since the SM contribution to ε_K is always positive, and vanishes for vanishing CKM phase, in order to obtain a conservative limit on SUSY contributions we set the CKM phase to zero, and allow the SUSY contribution to saturate the experimental value of ε_K.

We make use of the B-parameters given in eq. (3.3) and subtract one standard deviation to their central values, in order to extract a more conservative bound on SUSY parameters.

Barring accidental cancellations, following the procedure explained above, we obtain the limits on the δ-parameters reported in Tables 2–7. We give results for different values of the average squark mass, since the naive scaling of the constraints like $1/\tilde{m}$ is modified by the perturbative QCD corrections.

For Left-Right mass insertions, we consider two possible (extreme) cases: $|(\delta^d_{12})_{LR}| \gg |(\delta^d_{12})_{RL}|$ and $(\delta^d_{12})_{LR} = (\delta^d_{12})_{RL}$. In the second case, we combine the contributions proportional to $(\delta^d_{12})^2_{LR}$, $(\delta^d_{12})^2_{RL}$ and $(\delta^d_{12})_{LR}(\delta^d_{12})_{RL}$. This approach improves the one of refs. [5] and [7], where contributions proportional to $(\delta^d_{12})_{LR}(\delta^d_{12})_{RL}$ where considered independently from the ones proportional to $(\delta^d_{12})^2_{LR}$ and $(\delta^d_{12})^2_{RL}$. The interference between the various Left-Right terms, in the case $(\delta^d_{12})_{LR} = (\delta^d_{12})_{RL}$, produces a cancellation effect which is particularly sizeable at the NLO for $m_{\tilde{q}} = 500$ GeV, around $x = m^2_{\tilde{g}}/m^2_{\tilde{q}} = 1$, as one can see from Tables 4 and 5. This means that our results are not reliable for this particular case, since interference effects can change dramatically the final results for small variations of the input parameters.

On the other hand, the $(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}$ contribution can be treated independently from the $(\delta^d_{12})^2_{LL}$ and $(\delta^d_{12})^2_{RR}$ ones, since the latter ones do not generate Left-Right operators and are therefore suppressed with respect to the first contribution.

Let us compare our results with the previous analyses of refs. [6] and [7]. In both papers, the matrix elements were computed in the VIA, whereas we are using the B-parameters computed in lattice QCD. This allows us a consistent matching of the renormalisation-scheme and μ dependence of the QCD-corrected $\mathcal{H}^{\Delta S=2}_{\rm eff}$. Moreover we keep into account important non-perturbative effects present in the hadronic matrix elements.

If we use the same input parameters as in ref. [6], and neglect the SM contribution, we reproduce the results of ref. [6] for the QCD-uncorrected case. The values quoted in Tables 4-5 differ from the ones of ref. [6] due to the inclusion of the SM contributions in the present analysis and to the different choice of input parameters. As one can read from Table 4, our QCD-uncorrected and LO results are also different from those of ref. [7]. This is due to the different choice of the input parameters, to the neglect of the SM contribution in ref. [7] and to the choice of the scale μ. In particular, the results in Table 4 correspond to $\mu = 2$ GeV, while the authors of ref. [7] chose to evolve the $\mathcal{H}^{\Delta S=2}_{\rm eff}$ down to the scale μ' defined by $\alpha_s(\mu') = 1$. This choice may be questionable, since perturbation theory is expected to break down at such low scales.

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5. Constraints on R-parity violating couplings

In this section we update the constraints that can be derived from $\Delta S = 2$ processes on some R-parity violating couplings.

If R-parity invariance is not imposed, the superpotential may contain the following additional terms:

$$\lambda''_{ijk} u^c_i d^c_j d^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \lambda_{ijk} L_i L_j e^c_k \,, \tag{5.1}$$

where i, j and k are generation indices, and we omitted possible bilinear R-parity violating terms [23]. The λ_{ijk} and λ'_{ijk} are lepton number violating Yukawa couplings, while the λ''_{ijk} are baryon number violating ones. The λ_{ijk} are antisymmetric under the exchange of the first two generation indices, while the λ''_{ijk} are antisymmetric under the interchange of the last two flavour indices. We consider the R-violating couplings in the super-CKM basis.

Stringent constraints can be imposed on R-violating couplings by considering a variety of processes (see ref. [24] for a recent review on this subject). We concentrate on the limits that can be obtained from $\Delta S = 2$ processes, and in particular on the ones arising from Left-Right effective operators.

The most stringent constraints arise from the tree-level sneutrino-mediated contributions, that have the form

$$C_4(m_{\tilde{\nu}}) = \sum_i \frac{\lambda'_{i21}\lambda'^*_{i12}}{m^2_{\tilde{\nu}_i}} \,. \tag{5.2}$$

Additional constraints can be obtained by considering box diagrams with four R-violating couplings. As was pointed out in ref. [25], however, stronger constraints on the product of two λ' couplings can be obtained by considering box diagrams with the exchange of a slepton, together with a W or a $H^\pm$. We consider only contributions with top-quark exchange, since when light quarks are present in the loop the OPE is modified and the QCD evolution is much more involved. This computation is not available yet. In eq. (3) of ref. [25], the coefficient of the effective Hamiltonian at the electroweak scale was computed for $m_{H^\pm} = m_{\tilde{l}}$, by integrating out simultaneously all heavy particles. In order to avoid the appearance of large logs of the form $\ln m_t^2/m_{\tilde{l}}^2$ in the matching conditions, we consider this contribution for $m_{H^\pm} = m_{\tilde{l}} = 200$ GeV, and derive constraints on the product of the R-violating couplings $\lambda'_{i31}\lambda'^*_{i32}$.

Using the same procedure as that followed for R-conserving SUSY contributions, we obtain the constraints given in Table 8. As in the previous Section, we put the CKM phase to zero in order to obtain more conservative limits.

6. Conclusions

In our work we have provided an improved computation of the gluino-mediated SUSY contributions to K^0–$\bar{K}^0$ mixing in the framework of the mass insertion method. The improvement consists in introducing the NLO QCD corrections to the $\mathcal{H}_{\rm eff}^{\Delta S=2}$ [8] and in replacing the VIA B-parameters with their recent lattice computation [9]. As a glimpse at Tables 2–7 readily reveals, these improvements affect previous results in a different way, according to the operators of $\mathcal{H}_{\rm eff}^{\Delta S=2}$ one considers. The effect is particularly large for Left-Right operators.

We have also improved in the same way the constraints on some products of R-violating λ'-type couplings that can be derived from K^0–$\bar{K}^0$ mixing. Here the effects of QCD corrections, that were never considered before, are also very large, due to the Left-Right structure of the operator involved.

We have provided an analytic formula for the most general low-energy $\mathcal{H}_{\rm eff}^{\Delta S=2}$ at the NLO, in terms of the Wilson coefficients at the high energy scale. This formula can be readily used to compute ΔM_K and ε_K in any extension of the SM with new heavy particles.

The above results suggest several improvements for theoretical analyses. As for K-mixing, a computation of the Wilson coefficients at M_S is needed to have the complete NLO QCD-corrected and scheme-independent result. Another issue that needs to be clarified is the role played by the other classes of FCNC contributions which are present in SUSY in addition to the gluino exchange. In particular the chargino-squark contributions should be considered, although, as we said, for generic squark mass matrices, we expect the gluino exchange to set the correct size of the whole FCNC contributions.

Even more interesting is to extend this analysis to $\Delta B = 2$ processes and to combine the limits which can be derived by considering FCNC processes involving B and K mesons simultaneously. The lattice derivation of the B_B parameters is in progress and so we are confident to be able to perform a similar analysis in the B case.

FCNC and CP violating phenomena (in particular in B physics) are promising candidates for some indirect SUSY signal before LHC, and are in many ways complementary to direct SUSY searches. From this point of view the theoretical effort to improve as much as possible our precision on FCNC computations in a SUSY model-independent framework is certainly worth and, hopefully, rewarding.

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	NO QCD, VIA	LO, VIA	LO, Lattice B_i	NLO, Lattice B_i
x	$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LL}\rvert}$			
0.3	5.0×10^{-3}	5.7×10^{-3}	7.7×10^{-3}	7.7×10^{-3}
1.0	1.1×10^{-2}	1.2×10^{-2}	1.6×10^{-2}	1.6×10^{-2}
4.0	2.5×10^{-2}	2.9×10^{-2}	3.9×10^{-2}	3.9×10^{-2}
x	$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LR}\rvert} \qquad (\lvert(\delta^d_{12})_{LR}\rvert \gg \lvert(\delta^d_{12})_{RL}\rvert)$			
0.3	1.1×10^{-3}	8.4×10^{-4}	1.1×10^{-3}	9.6×10^{-4}
1.0	1.2×10^{-3}	9.3×10^{-4}	1.2×10^{-3}	1.1×10^{-3}
4.0	1.8×10^{-3}	1.3×10^{-3}	1.6×10^{-3}	1.5×10^{-3}
x	$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LR}\rvert} \qquad ((\delta^d_{12})_{LR} = (\delta^d_{12})_{RL})$			
0.3	2.0×10^{-3}	1.4×10^{-3}	8.9×10^{-4}	6.7×10^{-4}
1.0	1.1×10^{-3}	9.7×10^{-4}	1.8×10^{-3}	3.0×10^{-3}
4.0	1.3×10^{-3}	1.0×10^{-3}	1.4×10^{-3}	1.3×10^{-3}
x	$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$			
0.3	6.4×10^{-4}	3.9×10^{-4}	4.0×10^{-4}	3.3×10^{-4}
1.0	7.1×10^{-4}	4.4×10^{-4}	4.5×10^{-4}	3.7×10^{-4}
4.0	1.0×10^{-3}	6.1×10^{-4}	6.2×10^{-4}	5.2×10^{-4}

Table 2: Limits on $\mathrm{Re}\,(\delta_{ij})_{AB}\,(\delta_{ij})_{CD}$, with $A,B,C,D = (L,R)$, for an average squark mass $m_{\tilde q} = 200$ GeV and for different values of $x = m^2_{\tilde g}/m^2_{\tilde q}$.

	NO QCD, VIA	LO, VIA	LO, Lattice B_i	NLO, Lattice B_i
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LL} \rvert}$			
0.3	6.7×10^{-4}	7.5×10^{-4}	1.0×10^{-3}	1.0×10^{-3}
1.0	1.4×10^{-3}	1.6×10^{-3}	2.2×10^{-3}	2.2×10^{-3}
4.0	3.3×10^{-3}	3.8×10^{-3}	5.1×10^{-3}	5.1×10^{-3}
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LR} \rvert} \qquad (\lvert(\delta^d_{12})_{LR}\rvert \gg \lvert(\delta^d_{12})_{RL}\rvert)$			
0.3	1.5×10^{-4}	1.1×10^{-4}	1.4×10^{-4}	1.3×10^{-4}
1.0	1.6×10^{-4}	1.2×10^{-4}	1.6×10^{-4}	1.4×10^{-4}
4.0	2.3×10^{-4}	1.7×10^{-4}	2.2×10^{-4}	1.9×10^{-4}
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LR} \rvert} \qquad ((\delta^d_{12})_{LR} = (\delta^d_{12})_{RL})$			
0.3	2.6×10^{-4}	1.9×10^{-4}	1.2×10^{-4}	8.9×10^{-5}
1.0	1.5×10^{-4}	1.3×10^{-4}	2.4×10^{-4}	3.9×10^{-4}
4.0	1.8×10^{-4}	1.4×10^{-4}	1.8×10^{-4}	1.7×10^{-4}
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR} \rvert}$			
0.3	8.4×10^{-5}	5.2×10^{-5}	5.3×10^{-5}	4.4×10^{-5}
1.0	9.4×10^{-5}	5.8×10^{-5}	5.9×10^{-5}	4.9×10^{-5}
4.0	1.3×10^{-4}	8.1×10^{-5}	8.2×10^{-5}	6.8×10^{-5}

Table 3: Limits on $\mathrm{Im}\,(\delta_{ij})_{AB}\,(\delta_{ij})_{CD}$, with $A, B, C, D = (L, R)$, for an average squark mass $m_{\tilde{q}} = 200$ GeV and for different values of $x = m^2_{\tilde{g}}/m^2_{\tilde{q}}$.

	NO QCD, VIA	LO, VIA	LO, Lattice B_i	NLO, Lattice B_i
x	$\sqrt{\lvert\,\mathrm{Re}(\delta^d_{12})^2_{LL}\rvert}$			
0.3	1.4×10^{-2}	1.6×10^{-2}	2.2×10^{-2}	2.2×10^{-2}
1.0	3.0×10^{-2}	3.4×10^{-2}	4.6×10^{-2}	4.6×10^{-2}
4.0	7.0×10^{-2}	8.0×10^{-2}	1.1×10^{-1}	1.1×10^{-1}
x	$\sqrt{\lvert\,\mathrm{Re}(\delta^d_{12})^2_{LR}\rvert} \qquad (\lvert(\delta^d_{12})_{LR}\rvert \gg \lvert(\delta^d_{12})_{RL}\rvert)$			
0.3	3.1×10^{-3}	2.3×10^{-3}	2.8×10^{-3}	2.6×10^{-3}
1.0	3.4×10^{-3}	2.5×10^{-3}	3.1×10^{-3}	2.8×10^{-3}
4.0	4.9×10^{-3}	3.5×10^{-3}	4.4×10^{-3}	3.9×10^{-3}
x	$\sqrt{\lvert\,\mathrm{Re}(\delta^d_{12})^2_{LR}\rvert} \qquad ((\delta^d_{12})_{LR} = (\delta^d_{12})_{RL})$			
0.3	5.5×10^{-3}	3.3×10^{-3}	2.2×10^{-3}	1.7×10^{-3}
1.0	3.1×10^{-3}	2.7×10^{-3}	5.5×10^{-3}	2.8×10^{-2}
4.0	3.7×10^{-3}	2.8×10^{-3}	3.8×10^{-3}	3.5×10^{-3}
x	$\sqrt{\lvert\,\mathrm{Re}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$			
0.3	1.8×10^{-3}	1.0×10^{-3}	1.0×10^{-3}	8.6×10^{-4}
1.0	2.0×10^{-3}	1.1×10^{-3}	1.2×10^{-3}	9.6×10^{-4}
4.0	2.8×10^{-3}	1.6×10^{-3}	1.6×10^{-3}	1.3×10^{-3}

Table 4: Limits on $\mathrm{Re}\,(\delta_{ij})_{AB}\,(\delta_{ij})_{CD}$, with $A, B, C, D = (L, R)$, for an average squark mass $m_{\tilde{q}} = 500$ GeV and for different values of $x = m^2_{\tilde{g}}/m^2_{\tilde{q}}$.

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	NO QCD, VIA	LO, VIA	LO, Lattice B_i	NLO, Lattice B_i
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LL} \rvert}$			
0.3	1.8×10^{-3}	2.1×10^{-3}	2.9×10^{-3}	2.9×10^{-3}
1.0	3.9×10^{-3}	4.5×10^{-3}	6.1×10^{-3}	6.1×10^{-3}
4.0	9.2×10^{-3}	1.1×10^{-2}	1.4×10^{-2}	1.4×10^{-2}
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LR} \rvert} \quad (\lvert(\delta^d_{12})_{LR}\rvert \gg \lvert(\delta^d_{12})_{RL}\rvert)$			
0.3	4.1×10^{-4}	3.0×10^{-4}	3.8×10^{-4}	3.4×10^{-4}
1.0	4.6×10^{-4}	3.3×10^{-4}	4.2×10^{-4}	3.7×10^{-4}
4.0	6.5×10^{-4}	4.6×10^{-4}	5.8×10^{-4}	5.2×10^{-4}
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LR} \rvert} \quad ((\delta^d_{12})_{LR} = (\delta^d_{12})_{RL})$			
0.3	7.2×10^{-4}	4.4×10^{-4}	3.0×10^{-4}	2.2×10^{-4}
1.0	4.1×10^{-4}	3.5×10^{-4}	7.3×10^{-4}	3.7×10^{-3}
4.0	4.9×10^{-4}	3.7×10^{-4}	5.0×10^{-4}	4.7×10^{-4}
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR} \rvert}$			
0.3	2.3×10^{-4}	1.4×10^{-4}	1.4×10^{-4}	1.1×10^{-4}
1.0	2.6×10^{-4}	1.5×10^{-4}	1.5×10^{-4}	1.3×10^{-4}
4.0	3.7×10^{-4}	2.1×10^{-4}	2.2×10^{-4}	1.8×10^{-4}

Table 5: Limits on $\mathrm{Im}\,(\delta_{ij})_{AB}\,(\delta_{ij})_{CD}$, with $A, B, C, D = (L, R)$, for an average squark mass $m_{\tilde{q}} = 500$ GeV and for different values of $x = m^2_{\tilde{g}}/m^2_{\tilde{q}}$.

	NO QCD, VIA	LO, VIA	LO, Lattice B_i	NLO, Lattice B_i
x	$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LL}\rvert}$			
0.3	3.0×10^{-2}	3.5×10^{-2}	4.7×10^{-2}	4.7×10^{-2}
1.0	6.4×10^{-2}	7.4×10^{-2}	1.0×10^{-1}	1.0×10^{-1}
4.0	1.5×10^{-1}	1.7×10^{-1}	2.4×10^{-1}	2.4×10^{-1}
x	$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LR}\rvert} \qquad (\lvert(\delta^d_{12})_{LR}\rvert \gg \lvert(\delta^d_{12})_{RL}\rvert)$			
0.3	6.7×10^{-3}	4.7×10^{-3}	6.0×10^{-3}	5.3×10^{-3}
1.0	7.4×10^{-3}	5.2×10^{-3}	6.6×10^{-3}	5.9×10^{-3}
4.0	1.0×10^{-2}	7.2×10^{-3}	9.2×10^{-3}	8.2×10^{-3}
x	$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LR}\rvert} \qquad ((\delta^d_{12})_{LR} = (\delta^d_{12})_{RL})$			
0.3	1.2×10^{-2}	6.4×10^{-3}	4.5×10^{-3}	3.4×10^{-3}
1.0	6.7×10^{-3}	5.7×10^{-3}	1.3×10^{-2}	2.2×10^{-2}
4.0	8.0×10^{-3}	5.8×10^{-3}	8.0×10^{-3}	7.5×10^{-3}
x	$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$			
0.3	3.8×10^{-3}	2.1×10^{-3}	2.1×10^{-3}	1.8×10^{-3}
1.0	4.3×10^{-3}	2.4×10^{-3}	2.4×10^{-3}	2.0×10^{-3}
4.0	6.0×10^{-3}	3.3×10^{-3}	3.4×10^{-3}	2.8×10^{-3}

Table 6: Limits on $\mathrm{Re}\,(\delta_{ij})_{AB}\,(\delta_{ij})_{CD}$, with $A, B, C, D = (L, R)$, for an average squark mass $m_{\tilde{q}} = 1000$ GeV and for different values of $x = m^2_{\tilde{g}}/m^2_{\tilde{q}}$.

	NO QCD, VIA	LO, VIA	LO, Lattice B_i	NLO, Lattice B_i
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LL}\rvert}$			
0.3	4.0×10^{-3}	4.6×10^{-3}	6.2×10^{-3}	6.2×10^{-3}
1.0	8.4×10^{-3}	9.7×10^{-3}	1.3×10^{-2}	1.3×10^{-2}
4.0	2.0×10^{-2}	2.3×10^{-2}	3.1×10^{-2}	3.1×10^{-2}
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LR}\rvert}$ $\quad (\lvert(\delta^d_{12})_{LR}\rvert \gg \lvert(\delta^d_{12})_{RL}\rvert)$			
0.3	8.8×10^{-4}	6.2×10^{-4}	7.9×10^{-4}	7.1×10^{-4}
1.0	9.8×10^{-4}	6.9×10^{-4}	8.7×10^{-4}	7.8×10^{-4}
4.0	1.4×10^{-3}	9.6×10^{-4}	1.2×10^{-3}	1.1×10^{-3}
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LR}\rvert}$ $\quad ((\delta^d_{12})_{LR} = (\delta^d_{12})_{RL})$			
0.3	1.6×10^{-3}	8.5×10^{-4}	5.9×10^{-4}	4.5×10^{-4}
1.0	8.8×10^{-4}	7.6×10^{-4}	1.7×10^{-3}	2.9×10^{-3}
4.0	1.1×10^{-3}	7.7×10^{-4}	1.1×10^{-3}	9.9×10^{-4}
x	$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$			
0.3	5.0×10^{-4}	2.8×10^{-4}	2.8×10^{-4}	2.3×10^{-4}
1.0	5.6×10^{-4}	3.1×10^{-4}	3.2×10^{-4}	2.6×10^{-4}
4.0	8.0×10^{-4}	4.4×10^{-4}	4.5×10^{-4}	3.7×10^{-4}

Table 7: Limits on $\mathrm{Im}\,(\delta_{ij})_{AB}\,(\delta_{ij})_{CD}$, with $A, B, C, D = (L, R)$, for an average squark mass $m_{\tilde{q}} = 1000$ GeV and for different values of $x = m^2_{\tilde{g}}/m^2_{\tilde{q}}$.

	NO QCD, VIA	LO, VIA	LO, Lattice B_i	NLO, Lattice B_i
$m_{\tilde{\nu}}$ (GeV)	$\mathrm{Re}\,\lambda'_{i21}\lambda'^{*}_{i12}$			
100	1.4×10^{-10}	6.1×10^{-11}	6.3×10^{-11}	4.4×10^{-11}
200	5.8×10^{-10}	2.2×10^{-10}	2.3×10^{-10}	1.6×10^{-10}
500	3.6×10^{-9}	1.2×10^{-9}	1.3×10^{-9}	8.5×10^{-10}
$m_{\tilde{\ell}}$ (GeV)	$\mathrm{Re}\,\lambda'_{i31}\lambda'^{*}_{i32}$			
200	6.2×10^{-4}	2.4×10^{-4}	2.5×10^{-4}	1.7×10^{-4}
$m_{\tilde{\nu}}$ (GeV)	$\mathrm{Im}\,\lambda'_{i21}\lambda'^{*}_{i12}$			
100	3.1×10^{-12}	1.3×10^{-12}	1.4×10^{-12}	9.6×10^{-13}
200	1.2×10^{-11}	4.8×10^{-12}	4.9×10^{-12}	3.4×10^{-12}
500	7.8×10^{-11}	2.6×10^{-11}	2.7×10^{-11}	1.8×10^{-11}
$m_{\tilde{\ell}}$ (GeV)	$\mathrm{Im}\,\lambda'_{i31}\lambda'^{*}_{i32}$			
200	1.3×10^{-5}	5.2×10^{-6}	5.4×10^{-6}	3.7×10^{-6}

Table 8: Limits on $\lambda'_{i21}\lambda'^{*}_{i12}$ and on $\lambda'_{i31}\lambda'^{*}_{i32}$ from ΔM_K and ε_K for different values of the relevant SUSY masses.

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Erratum

1. Equation (2.7) should be replaced by the following expression:

$$a_i = (0.29, -0.69, 0.79, -1.1, 0.14)$$

$$\begin{aligned}
b_i^{(11)} &= (0.82, 0, 0, 0, 0), & c_i^{(11)} &= (-0.016, 0, 0, 0, 0),\\
b_i^{(22)} &= (0, 2.4, 0.011, 0, 0), & c_i^{(22)} &= (0, -0.23, -0.002, 0, 0),\\
b_i^{(23)} &= (0, -0.63, 0.17, 0, 0), & c_i^{(23)} &= (0, -0.018, 0.0049, 0, 0),\\
b_i^{(32)} &= (0, -0.019, 0.028, 0, 0), & c_i^{(32)} &= (0, 0.0028, -0.0093, 0, 0),\\
b_i^{(33)} &= (0, 0.0049, 0.43, 0, 0), & c_i^{(33)} &= (0, 0.00021, 0.023, 0, 0),\\
b_i^{(44)} &= (0, 0, 0, 4.4, 0), & c_i^{(44)} &= (0, 0, 0, -0.68, 0.0055),\\
b_i^{(45)} &= (0, 0, 0, 1.5, -0.17), & c_i^{(45)} &= (0, 0, 0, -0.35, -0.0062),\\
b_i^{(54)} &= (0, 0, 0, 0.18, 0), & c_i^{(54)} &= (0, 0, 0, -0.026, -0.016),\\
b_i^{(55)} &= (0, 0, 0, 0.061, 0.82), & c_i^{(55)} &= (0, 0, 0, -0.013, 0.018),
\end{aligned} \tag{6.1}$$

2. the following footnote should be added just before eq. (2.7):

   In the previously published version of this paper, the magic numbers in eq. (2.7) were erroneously given in the basis in of ref. [8, eq. (13)] instead of the basis (2.3) used in this work (we thank P. Slavich and F. Zwirner for pointing this out to us). However, all the numerical results in Section 4 and 5 of the previous version were correct, since they were obtained using the full evolution matrix instead of eq. (2.6).